Topic B: Life on Phorcys

Team 522

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Abstract

In this article, we investigate the possible forms of life that may exist on planet Phorcys. Our approach is to determine key parameters describing the planet: temperature variation, ice cover, surface pressure and ocean presence and depth. We find that Phorcys is covered in a global ocean, with large ice caps and the possibility of an ice free equatorial region. The presence and size of this region is determined by a combination of the salinity of the ocean water, which affects the freezing point of water, and the day length of the planet, which affects how temperature varies with latitude. earth levels of salinity and day length lead to an ice covered planet, but salinity levels tolerable by many known species lead to an ice free region. Alternatively, an ice free region is predicted if the day on Phorcys is three-quarters that of earth. After gaining this understanding of the climate on Phorcys, we investigate life. Our approach is to investigate life through the various layers of the deep oceans of Phorcys, which we find extend to 33 km, three times deeper than earths oceans. At 7.5 km on Phorcys we identify an unusual behaviour in the viscosity of water with implications for life at this depth. Additionally, we identify a depth bound of 22 km for organisms with a cell membrane similar to earth life by estimating the magnitude of pressure fluctuations at depth.

1 Overview

Phorcys is a Greek primordial sea god who governed over the "hidden dangers of the deep" [Ats11]. As will become apparent, this name for the new planet is rather appropriate.

The problem we face is to try to understand what kinds of life might exist on an earth-like planet in the habitable zone of a star, given that the mass of the planet is 8 times that of earth, the radius is twice that of earth, the surface temperature is 250 K, and a four-legged animal evolved to live near the equator of the planet. We wish to characterize the regimes in which earth-like life could exist and how that life would adapt to the conditions on the planet.

Our strategy will be to first establish a quantitative picture of Phorcys; we wish to understand its composition, surface pressure, and the variation of temperature across the planet. To do this, we will use the parameters that we know – mass, radius, mean temperature – and make inferences on how unknown parameters likely relate to earth's parameters.

Once we understand the conditions on the planet, we will inspect which areas of Phorcys appear similar to areas of earth in order to understand how life might be similar or different. Next, we estimate limits at which various types of life as we know it would be unable to exist and speculate as to what kind of adaptions would allow other life. We finally attempt to imagine what sorts of animals would thrive in such an environment.

Contents

2 Quantitative model

2.1 Assumptions

In order to proceed, we've made several assumptions. Each are justified as they are applied.

1. Phorcys is fully differentiated and is composed of an iron core, silicate layer, and has the possibility of having a water-rich layer
2. The ratio of the mass of the atmosphere to the mass of the Phorcys is the same as for earth
3. The light flux from nearby stars is comparable to the light flux incident upon the earth
4. The day length on Phorcys is similar to the day length on earth

2.2 Estimation of planet's surface pressure

Notation: We will use the subscript e to denote earth parameters; in particular, M_e is the mass of the earth and R_e is the radius of the earth. Similarly, the subscript p will denote parameters for Phorcys.

Following the approach of [Sea11, p. 445], we estimate the surface pressure as simply force per area; the force is $g_p m_{a_p}$ where g_p is the gravitational constant for Phorcys, and m_a is the mass of the atmosphere, and the area is just $4\pi R_p$, where R_p is the radius of Phorcys.

That is, the pressure at the surface, $P_{p,s}$ is given by

$$P_{p,s} = \frac{g_p m_{a_p}}{4\pi R_p^2}$$

We can calculate the gravitational parameter:

$$g_p = \frac{GM_p}{R_p^2} = \frac{G \cdot 8M_e}{(2R_e)^2} = 2g_e$$

where G is Newton's constant and g_e is the gravitational parameter of earth.

We can estimate m_{a_p} in terms of m_{a_e}, the mass of the atmosphere on earth. Specifically, we assume the ratio of the mass of the atmosphere to the mass of the planet, m_a/M, is the same for the earth and for Phorcys. Seager notes that "higher-mass planets are more likely to retain more massive atmospheres against progressive loss", however this relationship also depends heavily on the planets history and development. Venus, specifically, has a run-away greenhouse effect and subsequently an atmospheric mass 100 times that of earth's, while itself having slightly less mass. Phorcys has a temperature comparable to earth's, which indicates that at least no run-away greenhouse process occurred. Additionally, the presence of a four-legged animal suggests, although does not necessitate, that the atmosphere is comparable to earth's in its composition.

With this assumption we have that

$$\frac{m_{a_e}}{M_e} = \frac{m_{a_p}}{M_p} \implies m_{a_p} = \frac{M_p}{M_e} m_{a_e} = 8m_{a_e}$$

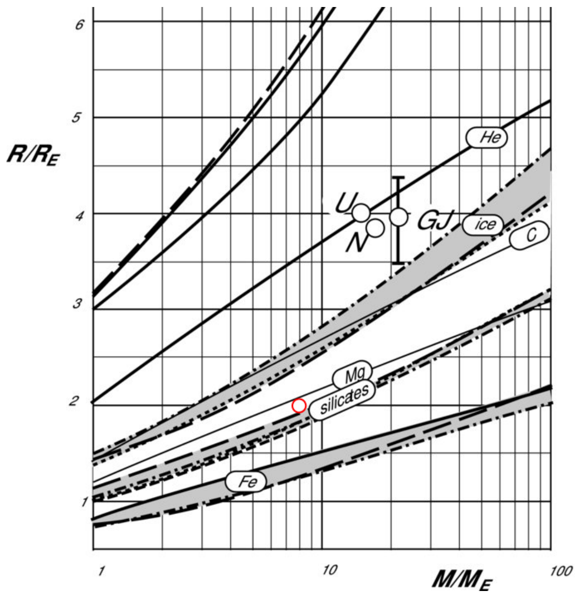
This allows us to estimate the $P_{p,s}$, the surface pressure on Phorcys:

$$P_{p,s} = \frac{g_p m_{a_p}}{4\pi R_p^2} = \frac{2g_e \cdot 8m_{a_e}}{4\pi(2R_e)^2} = 4P_{e,s}$$

That is, the surface pressure on Phorcys is about four times the surface pressure on earth.

2.3 Planet composition

The radius and mass can be used to estimate the composition of Phorcys. Grasset et al modelled terrestrial exoplanets to try to determine planet composition based on the mass and radius ([GSS09]). This model assumes that the planet is fully differentiated (i.e. the constituent elements have had the time to separate), which is reasonable to assume because there has been time for a four-legged animal to evolve. Additionally, the model assumes the planet has an iron core, a silicate layer, and possibly a water-rich layer. These assumptions also seem reasonable, as Phorcys has the same density as earth and a comparable average temperature, indicating a composition roughly similar to earth. As Figure 1a shows, under these assumptions, Phorcys has a significant amount of water in its composition; furthermore, Figure 1b indicates that the amount of water is close to 20% of the planet, by mass.



(a) This is Figure 9 of Grasset et al's work (truncated at 6 R/R_e); the original caption was:"M-R relationships for solid planets composed of iron, silicates, and ices (shaded areas). The amount of water varies from 0% to 100% from bottom to top of the white domain limited by the silicates curves at the bottom and the pure ices curve at the top (see Figure 4). The results from previous studies and for several pure compounds (H, He, C, Fe, Mg) are also plotted for comparison. Position of GJ436b and solar gaseous planets are indicated by open circles." [GSS09, Figure 9]
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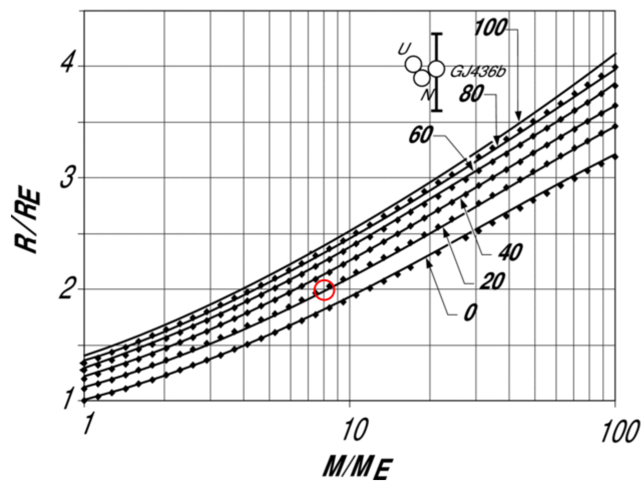

(b) This is Figure 4 of Grasset et al's work; the original caption was: "M-R relationships below 100 ME in the reference case. Numbers indicate the amount of water in wt% (Xw). Black diamonds: model results from Equation (2). Black lines: M-R relationships using Equations (3) and (4) with parameters from Table 2. Uranus, Neptune, and GJ436 B are indicated by open circles." [GSS09, Figure 4]
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Figure 1: Red circles have been added to guide the eye towards the point representing Phorycs's parameters $(R_p, M_e) = (2R_e, 8M_e)$ (note that the size of the circle has no physical meaning; we were given exact values). Grasset et al modelled exoplanets with mass from 1 earth-mass to 100 earth masses and various compositions of water. Figure 1b shows different regimes of composition for different radii and masses. Phorcys, with $M_p/M_e = 8$ and $R_p/R_e = 2$ follows into the white domain of planets composed of between 0% and 100% water. In particular, we conclude that our planet is not a gas giant nor an ice planet. This motivates inspection of Figure 1b, in order further pin down the water content of the planet. We can see that Phorcys falls very near to the 20% water curve. Grasset et al cite 4.5% as the uncertainty in these curves for planets fitting the assumptions, so Phorcys likely has a very significant amount of water.

2.4 Water depth

We can estimate the depth of water on Phorcys. We've assumed Phorcys is fully differentiated into layers: a core of iron, a silicate layer, and then a water layer.

So the depth of water, h, is the difference between two radii: R_p, the radius of Phorcys, and R_w, the radius from the center to the silicate/water transition.

We can approximate the density of water as a constant, $\rho_w = 1000$ kg/m^3, and denote the average density of the rest of the planet as ρ_r. Then

$$\frac{4}{3}\pi(R_p^3 - R_w^3)\rho_w = 0.2M_p \qquad \& \qquad \frac{4}{3}\pi R_w^3\rho_r = 0.8M_p$$

It is straightforward to solve this system of equations for the two unknowns R_w and ρ_r, yielding the depth of water as $h = R_p - R_w = 5506$ km.

2.5 Pressure induced phase transition

Around 0° C, water turns to ice by approximately P_{ice} =6,300 atm ([Cha13b]). We can then make a rough calculation for at what depth the water on Phorcys turns to ice: taking $\rho_w \approx 1000$ kg/m^3.

$$P_{\text{ice}} = 2\rho_w g_e h \implies h = \frac{P_{\text{ice}}}{2\rho_w g_e} \approx 33 \text{ km}$$

Even accounting for the compressible nature of water the density changes to at most 1200 kg/m^3 ([Cha13b]). Plugging in this value we get $\approx$ 27 km. Note that we have neglected the 4 atm pressure contributed from the atmosphere, but it is clear this is negligible.

2.6 Water viscosity

Interestingly, water has a local minimum in viscosity around 150 MPa or about 1500 atm (see Figure 2), which, by very similar calculations as above, occurs around 7.5 kilometers below the surface. Again we have neglected surface atmosphere in this calcultion.

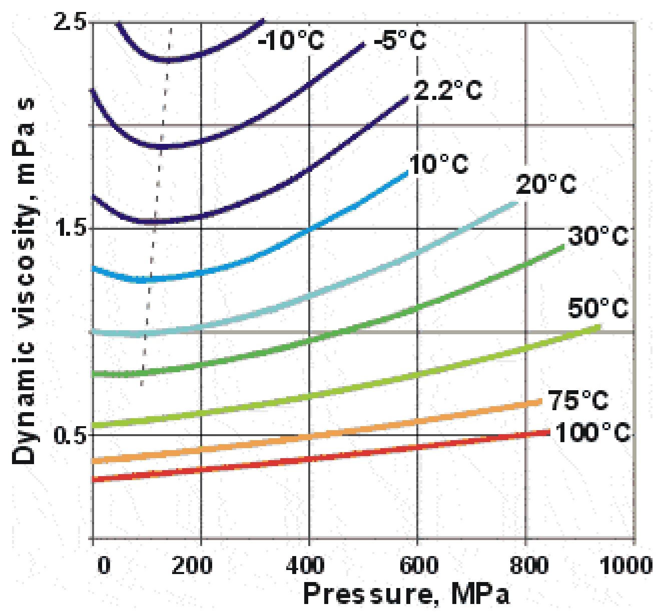


Figure 2: Water viscosity versus temperature; plot taken from [Cha13a].

2.7 Surface temperature variability

We know that the mean surface temperature is $T_0 = 250$ K. It is plausible that the temperature is significantly above this value near the equator, so we would like to understand how temperature varies on the surface of Phorcys with latitude.

To do so, we model the surface temperature variability using the diffusion equation with a driving term:

$$c\frac{\partial T}{\partial t} = \vec{\nabla}\cdot(D\vec{\nabla}T) + S(\varphi) - I(\varphi)$$

Several physical constants appear here, as well as the functions $S(\varphi)$ and $I(\varphi)$. We approach each of these terms below by finding their earth values, then arguing that they are the same as earth values or finding how they should relate to earth values. We begin by looking at D, the diffusivity of the planets surface. Since Phorcys is a water covered planet the dominate effect redistributing heat is the presence of ocean current. From [Sea11, p. 508] we have that:

$$D = L_{\text{eddy}}^2 \frac{g}{N\theta_0}\frac{\partial\bar{\theta}}{\partial y}$$

Where L_{eddy} describes the length scale of circulation currents on our planets, N is the Brunt-Väisälä frequency, and θ_0 is the temperature at the equator. The partial derivative of $\bar{\theta}$ describes fluctuations of temperature in the atmosphere. We cannot hope to pin down a theoretical value for each of these terms on Phorcys: this is not even achieved in studies of earth. Instead, we will find the g dependence of this expression and relate the value of D on Phorcys to a fitted value for earth. Beginning with L_{eddy} we have [Gil82, pg 662]:

$$L_{\text{eddy}} = \frac{(gz)^{1/2}}{f}$$

Where z is a height above the surface and f the Coriolis parameter, which is proportional to the rate of rotation of the earth. As well, we have that:

$$N = \left(\frac{-g}{\rho}\frac{\partial\rho}{\partial z}\right)^{-1/2}$$

Taking the surface density (of water) to be similar to that on earth, it remains now only to estimate the dependence of $\frac{\partial\rho}{\partial z}$ on g. We take the water to react to a pressure linearly with force, and consider a small cube of water within the ocean. Then taking the pressure on the cube to be due to the weight of water above the object, a simple calculation gives that:

$$\frac{\partial\rho}{\partial z} \;\propto\; g$$

Substituting this and the expressions for L_{eddy}, N into D and keeping only the g and f dependence we have:

$$D \;\propto\; \frac{g}{f^2}$$

And so $D_p = 2D_{\text{e}}\left(f_e/f_p\right)^2$, an important result we will make use of shortly. The term $I(\varphi)$ represents the emission of energy from the surface of Phorcys. This is given by the Stephan-Boltzmann law:

$$I(\varphi) = \varepsilon\sigma T(\varphi)^4 \tag{1}$$

Because both earth and Phorcys are largely covered in water, we take ε, the emissivity, to be that of earth, where $\varepsilon = 0.83$ [Nor75]. We take the temperature variations to be small compared to the equilibrium temperature, T_0, and linearize the Stephan-Boltzmann law around T_0, yielding:

$$I(\varphi) = \varepsilon\sigma T_0^4 + \varepsilon\sigma T_0^3 T(\varphi) \tag{2}$$

Note that the coefficients here may be calculated from the value of $T_0 = 250$ K for Phorcys.

$S(\varphi)$ is the flux incident on the earths surface from both the sun and interior sources. Interior sources we take to be isotropic and contribute a constant term labeled S_0, while the parent stars radiation follows a sine function in the azimuthal angle, giving:

$$S(\varphi) = S_0 + K(1-\lambda)\sin(\varphi) \tag{3}$$

Where K is the stellar flux incident on a surface perpendicular to the line of sight between the planet and star, and A represents the fraction of stellar radiation reflected from the surface of the object. Again using the fact that Phorycs and earth have similar surfaces, the constant λ is taken to be the same as that of earth: $\lambda = 0.68$ [Nor75]. Substituting these expressions into the diffusion equation we find the differential equation:

$$0 = \frac{D}{r^2}\nabla^2 T + S_0 + K(1-\lambda)\sin(\varphi) - \varepsilon\sigma T_0^4 - \varepsilon\sigma T_0^3 T \tag{4}$$

Rewriting this differential equation in polar coordinates we find that:

$$0 = \frac{D}{r^2}\frac{\partial}{\partial\varphi}\left(\sin(\varphi)\frac{\partial T}{\partial\varphi}\right) + S_0 + K(1-\lambda)\sin(\varphi) - \varepsilon\sigma T_0^4 - \varepsilon\sigma T_0^3 T \tag{5}$$

Where T is assumed to be symmetric about the polar axis (the dominant temperature variations are across latitudes). We impose that the North and South poles have the same temperature, giving the boundary condition $T(0) = T(\pi)$.

Two parameters remain free in this differential equation, S_0 and D. North fitted his model to earth data and found that $D/Br_e^2 = \alpha$ where $\alpha = 0.6$ and r_e the radius of earth. B is a factor in his own expansion $I = A_e + B_e T$ (which is about a different point than ours). Then $D_{\text{earth}} = a^2 B\alpha$, and using that $D_p = 2D_e\left(f_e/f_p\right)^2$ from above, we have $D_p = 2r_e^2 B\alpha\left(f_e/f_p\right)^2$. The actual parameter appearing in the differential equation is $b_p = D/r^2$, so for earth:

$$b_e = \alpha B_e \tag{6}$$

And for Phorcys, with $r_p = 2r_e$:

$$b_p = \frac{\alpha}{2}B_e\left(f_e/f_p\right)^2 \tag{7}$$

A measured value of $B_e = 1.45Wm^{-2}C^{-1}$ is given in [Nor75], allowing us to compute b_e or b_p explicitly. We are left then with S_0 and $\left(f_e/f_p\right)^2$. S_0 is difficult to estimate, but fortunately the value of $T_0 = 250$ K can be used to determine S_0. Our procedure is to numerically solve the diffusion equation to find $T(\varphi)$ and to compute T_0 from this distribution. S_0 is then optimized to yield $T_0 = 250$ K.

The results of this analysis are presented in Figure 3, and the MATLAB implementation is presented in Appendix A. As we do not have any data regarding the angular frequency of Phorcys, we plotted the Temperature versus latitude curve for several values of f_e/f_p. We found that frequencies somewhat higher than on earth give temperatures at the equator consistent with liquid surface water, which is suggested as likely to exist based on the presence of a four legged creature.

In Figure 4 we plot the temperature curve of Phorcys on the same plot as earth for comparison, where Phorcys is taken to have the same frequency of rotation of earth. We can note that a larger difference exists between pole and equator on phorcys than on earth. This is sensible, as both have a similar driving radiation but Phorcys a lower mean temperature than earth.

We can gauge the success of our model (Figure 4) by comparing our earth plot to empirical data on earth (Figure 5). The equatorial temperatures of the model and empirical data are within a few degrees Celsius, while the model's North pole is about 8 degrees higher temperature, and the model's South pole is about 40 degrees higher.

The model assumes that both planets are entirely covered in liquid water; while water currents can run under ice, the earth has large landmasses which block the flows. Thus, it's likely that our model is more accurate for Phorcys than for earth. This effect explains the discrepancy between our earth model and experimental data in the far southern latitudes.

The fitted value of S_0 for Phorcys is $S_0 = 207\ \text{Wm}^{-2}$, and for earth $S_0 = 458\ \text{Wm}^{-2}$. These results compare unfavorably with our initial interpretation of S_0 which was the heat flux due to internal sources. On earth, this is measured to be $0.087\ \text{W m}^{-2}$, a discrepancy of four orders of magnitude. This identifies a weakness in our model, and suggests an improved model would incorporate a more realistic function

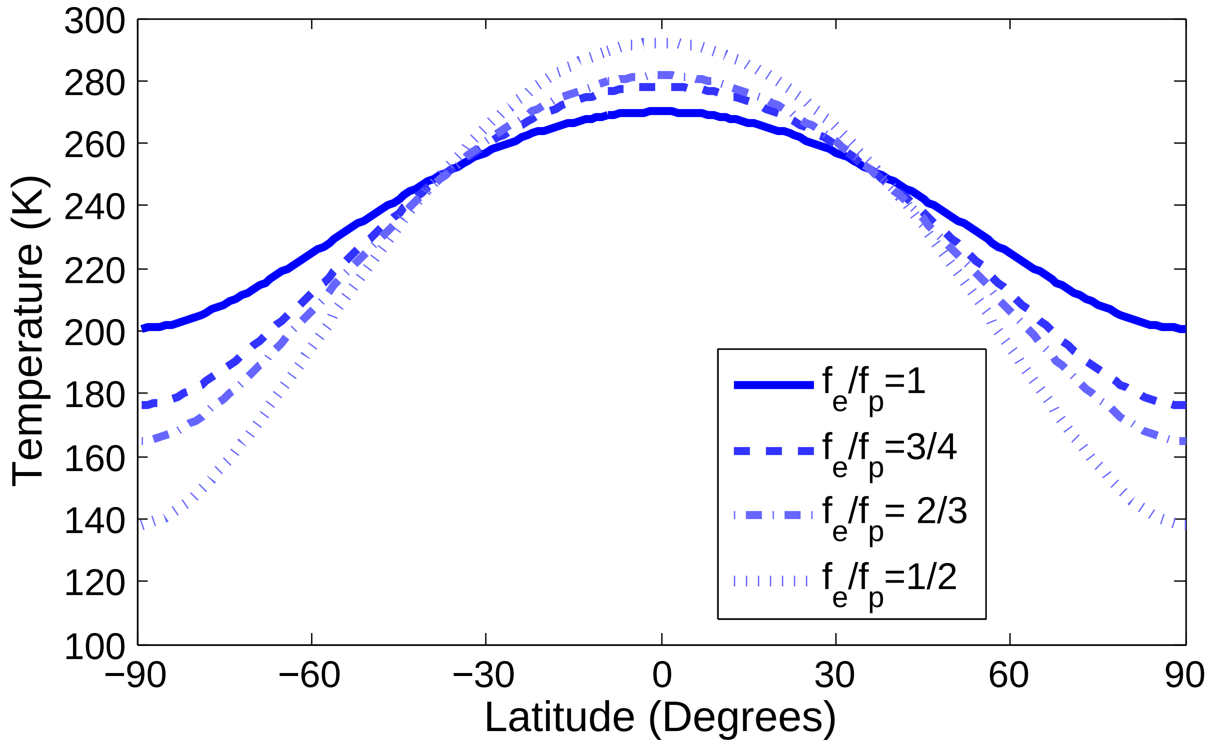


Figure 3: Plot of modelled temperature on Phorcys as a function of latitude. The ratio of f_e/f_p can be viewed as the length of a day on Phorcys in units of earth days. Here we model temperature for Phorcean day length ranging from 1/2 of an earth day to a full earth day. The model is quite sensitive to day length as diffusivity goes like the square of this ratio so the large variation between these lines is to be expected.

$S(\varphi)$. On earth for instance the tilt of the axis of rotation relative to the sun introduces complications to $S(\varphi)$, partly explaining the large constant term, because some of the flux is smeared over the planet (since we average latitudinally).

For this reason we drop this interpretation of S_0 and instead view it simply as a parameter introduced into the model in order to allow the given value of T_0 to be matched. In this context, adding a constant term to the incoming flux seems the simplest way mathematically to impose a given T_0.

2.8 Ice caps

By calculating the freezing point of water at different salinities and comparing to the modelled temperature of Phorcys at different latitudes, we can estimate where, on average, ice would form on Phorcys (see Figure 6). The freezing points were calculated as per [Une83].

While the freezing points we calculated are in terms of salt content, freezing point depression as a phenomenon mostly depends on the density of particulate in the liquid, and not the specific molecular composition. It's reasonable to assume, therefore, that some particulate might exist in Phorcys's oceans to depress the freezing point.

In order to determine where ice caps form, we would need to know the molar density of the particulate (to determine the freezing point) and the variation of temperature at each point (to find out if the temperature stays low enough for long enough). On earth, the ice caps begin around at about 263 K, or 10 degrees below the predicted freezing point [Nor75]; it seems unlikely that some phenomenon would increase the freezing point on Phorcys.

Of course, we can't determine the particulate density in Phorcys's oceans. We can, however, note

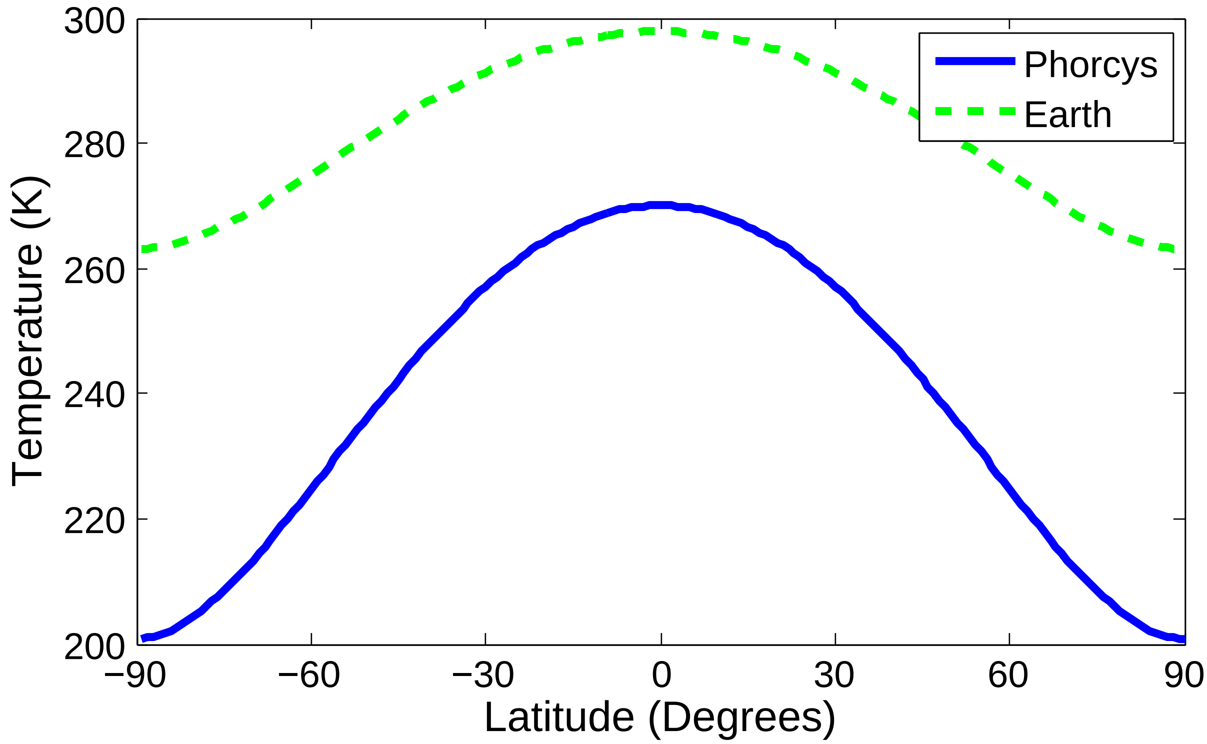


Figure 4: Plot of modelled temperature versus latitude for earth and Phorcys.

that there are types of osmoregulating fish on earth that can survive 120 parts per thousand of salinity, despite being evolved in earth's oceans, with a salinity of 35 parts per thousand [LBC+04].

If Phorcys had no particulate in its water, and the temperature variation was not enough to counteract freezing, all of the oceans would freeze. It seems unlikely that life would develop on this frozen ball without liquid water. We know a four-legged animal evolved on Phorcys near the equator, so we assume there is at least the relatively small amount of particulate to depress the freezing point there to around earth's 35 ppt. Since life, even on earth, is capable on surviving under higher-salinity conditions, it's certainly plausible that higher concentrations of particulate could exist; in this case, liquid water would probably exist in the latitudes with average temperature above the associated freezing point.

2.9 Summary of quantitative model

From the radius, mass, average surface temperature, and presence of a four-legged animal, and the assumptions outlined earlier we've found that:

1. The surface pressure is about four times that of earth's
2. The planet is composed of about 20% water, in comparison to earth's nearly 0% [Sea11]; the vast majority of that water is frozen, but oceans twice as deep as earth's cover the planet.
3. The surface temperature ranges from 200 K at the poles up to 270 K at the equator with a mean temperature of 250 K.
4. Ice caps likely cover the poles but not the equator

These quantitative parameters help paint a qualitative picture of the planet; in comparison to earth, our planet has high pressure, enormous amounts of water, and cold temperatures.

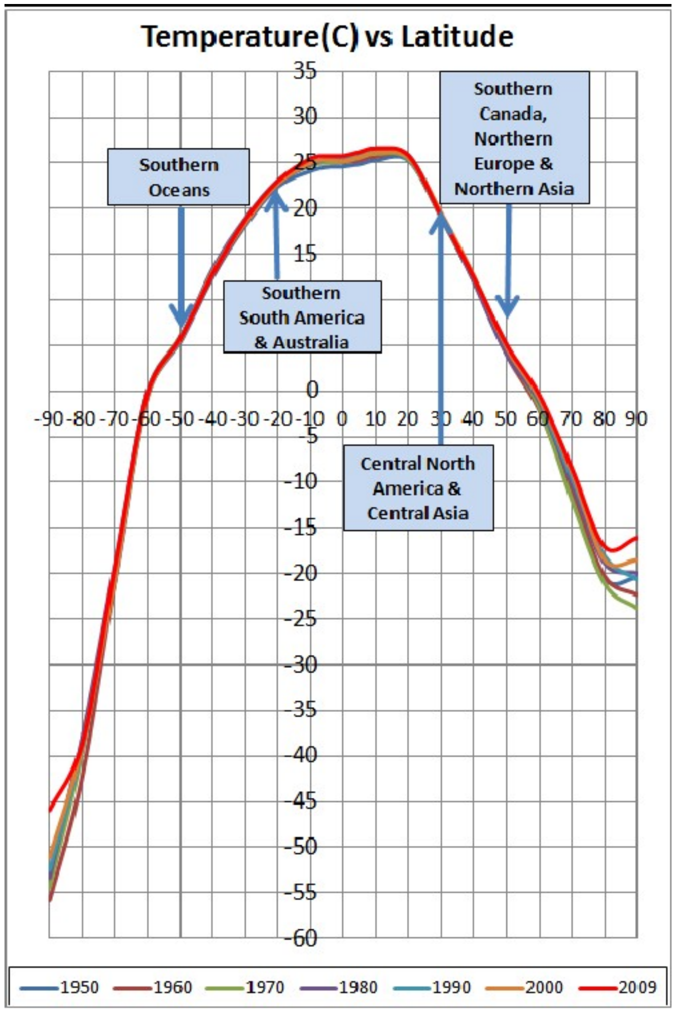


Figure 5: Empirical plot of temperature versus latitude on earth.

3 Life

Now that we have obtained estimates of a few parameters describing conditions on Phorcys, we are in a better position to understand how life functions on Phorcys. Our starting point is the discovery that Phorcys is composed largely of water, and life there must be suited to a ocean covered world.

As a first step towards understanding what forms of life may exists on Phorcys, we look to Figure 4. The variation of earth and Phorcys temperatures with latitudes allow equatorial regions on Phorcys to have temperatures overlapping with earths polar regions. Analysis of the earth and Phorcys temperature curves gives an overlap with earth temperature in the $(-40°, 40°)$ latitude, or 65% of the total surface area. Earth animals that live at such extreme temperatures include most Antarctic species such as penguins, walruses, seals, and so on, as well as several northern species. Polar bears for instance are known to regularly frequent 88° north. This comparison to earth suggests a wide variety of life exists on Phorcys.

Since Phorcys is an ocean planet, we divide our discussion of it life according to the depth at which it lives, beginning with surface inhabitants.

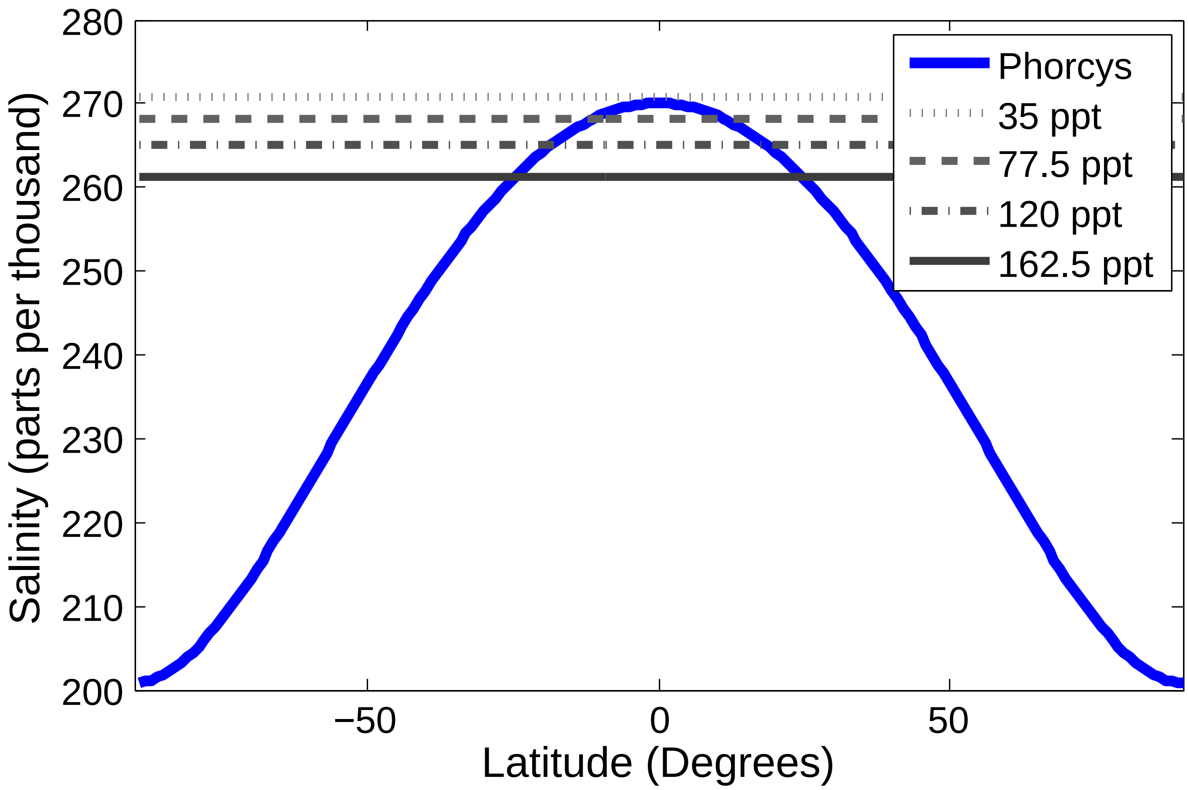


Figure 6: Plot of modelled temperature versus latitude for Phorcys, with water freezing points at different salinities (measured in parts per thousand) indicated. 35 ppt is the average salinity in earth's oceans.

3.1 Surface Life

One interesting question concerns the existence of four-legged creatures on Phorcys. Although a water planet, we have identified two scenarios allowing solid masses where four-legged creatures may have evolved. The first is inspired by polar bears, who spend many months per year at sea. Similar creatures may have evolved to live on the ice, at edge of the liquid region of Phorcys.

A second scenario is inspired by the coral reefs of earth. Corals are composed of the remains of many small organisms that have bonded together. A similar structure, but floating, could exist on Phorcys. Inhabiting such a "raft" could provide an advantage to sea creatures, for example by allowing escape from predators. Rafts suited to live at the equator where temperatures are over 260 K are unlikely to also be suited to the polar temperatures of 200 K. Thus a necessary condition for the formation of such rafts is the existence of stable regions in the ocean currents.

That the current will be zero somewhere on the ocean world of Phorcys is given by the "Hairy Ball Theorem" [Eis], a mathematical result from algebraic topology which states that there is no non-vanishing vector field of a sphere of even dimension. Applied to Phorcys, this means the ocean currents must vanish *somewhere*. Additional support for the existence of a region stable for the formation of rafts can be seen in the Sargasso sea, a region in the mid-latitude Atlantic which ocean currents fully circle, and where interior ocean currents drop to zero [OA13]. Plants such as the seaweed from the genus Sargassum collect in this region due to these peculiar ocean currents. The large amount of seaweed in the region attracts young sea turtles, who make use of the seaweed as cover from predation until they have matured.

With these facts in mind, such floating rafts collected in a stable region seem possible, and could support surface-based life.

We hypothesize that surface animals on Phorcys would have shorter limbs to minimize surface area and hence heat loss to the very cold environment. The colder temperatures may also encourage gigantism, as larger animals with a smaller surface area to body ratio tend to be more common in colder climates.

The higher air pressure, and consequent higher pressure of oxygen, would allow more efficient oxygen exchange in the animals lungs. This would allow smaller lungs in comparison to the size of the animals. Lung size is a limiting factor in the size of animals as it becomes difficult to move air throughout a large lung. The importance of this effect is apparent in the largest animals to have existed on earth, dinosaurs, which used an avian-like lung structure consisting of many smaller lungs to overcome this difficulty. Thus the higher pressure on Phorcys, as well as the colder temperatures, encourages gigantism.

A counteracting factor is the greater gravitational constant. This increases the energy cost of moving blood through the body, especially to elevated structures such as long necks and elevated heads. Such structures then seem unlikely on Phorcys, contributing further to the shorter limbs already encouraged by colder temperatures.

On the ice caps, we would expect no plants to grow; however, floating plants and cyanobacteria in the vast oceans would help oxygenate the planet.

3.2 Subsurface zone: 0 - 200 metres

Animals will have to deal with much higher pressures. For a static fluid the pressure is

$$P = \rho g h$$
$$\Rightarrow P = 2\rho_w g_e h$$

So the pressure under water will increase twice as fast as it does on earth. Additionally since the surface pressure is 4 atm we have an additional 3 atm of pressure under water when compared to earth. This is the fundamental difference between this layer of the ocean and the same region on earth.

The region above 200 meters is the Photic zone and will have about 90% of the marine life. The bottom of this region will have only 1% of the light that was on the surface so all photosynthesis must happen here. Since we have assumed similarities in the stars of Phorcys and earth the light in this region should be similar on both planets.

Plants on earth grow at a wide variety of depths so we would not expect the pressure to be a constraint on the flora. However there can be no roots since the bottom of the ocean is a bed of ice 30 km below. This restricts the plant life to vegetation that float on the surface of the water. On earth floating varieties of ferns and even seeded plants exist, it could be expected that similar plants will exist on the surface of Phorcys.

The cyanobacteria on earth can be found in almost every aquatic climate, there is no reason to expect that they would have to change to live on the surface of the equatorial waters of Phorcys. On earth cyanobacteria was responsible for the rise of oxygen in the atmosphere. Since it is so adaptive it is plausible that a similar event would have occurred on Phorcys.

On earth pressure increases about 1 atm for each 10 m increase in depth so the extra atmospheric pressure can be thought of as an extra 30 m of water. Recalling that we have no land on Phorcys we will expect our fish in this zone to be more like the oceanic fish of earth. There are relatively few earth species that live their entire lives far from the coast. The fish living in the deep ocean may move vertically more than 100 m in a day [tun99]. With this in mind an incremental 30 m has no noticeable effect. The more interesting effect due to the increased air pressure will be the increase of oxygen dissolved in the water. This leads to changes in fish. For the oceanic fish like Tuna we could expect them to have increased maximum and average aerobic output. This would lead to moving through the water more quickly with access to the extra oxygen.

We can conclude that life above 200 m on Phorcys will be much the same as deep ocean life above 200 m on earth. The more quickly increasing pressure and the surface pressure will not greatly affect our fish as oceanic fish in this zone on earth routinely experience these types of pressures. The largest change in these oceanic fish will be due to the increased oxygen. This change will continue through the entire ocean.

3.3 200 – 4000 metres

At these depths, we expect creatures to resemble earth's oceans beyond the Photic region. The pressures are comparable to pressures on earth; beyond the reach of sunlight, the temperatures will likely be comparable, both close to 0° C. We would expect less vertical mobility in the creatures because the pressure changes twice as fast as on earth, which could prevent fish from adopting the same methods as some fish on earth. In this vein, we would expect fewer fish to adopt the swim bladders that some earth fish use to rapidly move vertically; the energy expended in maintaining a cavity quickly becomes untenable, even on earth. Like on earth, we would expect biolumininesence to be a common adaptation to the lack of light.

3.4 4000 – 20 000 metres

Fish have only been found to 8400 meters under the water on earth, according to [Rya12], and pressure is the suspected reason for the drop off. The same pressure on Phorcys is around 4000 meters, so we would suspect earth-like fish to no longer be present at this depth. Creatures at these depths would likely be small and gelatinous with minimal bone structure, like jellyfish; the pressure is to deep great otherwise. earth's oceans reach around 11 km; at these depths, not much is known. When cameras were dropped in the Mariana trench, they found 4–10 inch long single-celled creatures as well as jellyfish [dro11]. Lacking some new material to construct organisms, it seems likely that similarly gelatinous, boneless creatures would inhabit these depths on Phorcys.

One interesting aspect on this regime is that around 7.5 km, the water viscosity reaches a minimum (see section 2.6). On earth, the minimum would be at 15 km (below the deepest parts of earth's oceans), so this phenomenon doesn't have an earth analog. It seems very plausible that the method of locomotion of these gelatinous creatures could change; jellyfish on earth move in a complicated way that we are still understanding, which involves the drag from the water [Boy13]. As the viscosity begins increasing with depth instead of decreasing, the gelatinous inhabitants might find a different form of locomotion more favorable. There also could be types of motion that are only energetically favorable at the very lowest viscosity.

3.5 20 000 – 33 000 metres

The interior pressure of deep sea organism adjusts to match the exterior pressure. Organisms that live in the deep sea are not crushed by the pressure, and their adaptations revolve more around the lack of sunlight and low temperature conditions. Eventually, the pressure of water becomes so high that a phase transition occurs. At some depth water turns to ice and we expect life cannot exist, on Phorcys this occurs at 33000 metres, providing a first limit on the depth at which life might be found. It is interesting however to ask if there is an earlier limit to the extent of life: does life on Phorcys extend down to the ice-water barrier?

Every thermodynamic variable fluctuates. In particular, the pressure experienced by a cell in water fluctuates, creating pressure differences at different places on the cell. Thermodynamics tells us:

$$\langle (\Delta P)^2 \rangle = -T \left(\frac{\partial P}{\partial V} \right)_S \tag{8}$$

We can make use of the reciprocal relation to rewrite the partial fraction appearing here, and write this in terms of the isentropic compressibility:

$$\beta_S = \frac{-1}{V} \left(\frac{\partial V}{\partial P} \right)_S \tag{9}$$

The partial derivative of pressure with respect to volume appearing here increases as the pressure increases. If it does so significantly, there may be a pressure at which thermodynamic fluctuations in pressure create difference across the cell membrane to strong for the membrane to support. We will make an estimate of the depth at which this effect becomes important.

The isentropic compressibility may be related to the isothermal compressibility via $\beta_S = \beta_T/\gamma$, where *gamma* is the heat capacity coefficient (familiar from $PV^\gamma =$ constant, for instance). This is practical, as experimentally measured values of β_T for a wide range of pressures are available [FM]. Then rewriting $\langle(\Delta P)^2\rangle$ in terms of β_T we find that:

$$\langle(\Delta P)^2\rangle \ \propto \ \frac{1}{\beta} \tag{10}$$

To obtain an estimate of β_T we can fit a linear function $\beta_T(P) = \beta_{surf} - \alpha P$ to the values given in [FM]. Measurements were taken of the velocity of sound in water, and from these the authors compute β_T. The fitting procedure and data is shown in Appendix B. The obtained function for β_T is:

$$\beta(P) = 51.7596 - 0.0112P \tag{11}$$

To estimate the pressure differences a cell can support, we use measured values for the osmotic pressure of red blood cells. This is roughly 0.03 atm [DdLPM]. We denote the burst pressure of the cell p_b and let $p_b = \lambda p_s$, for α a parameter describing the strength of the deep sea cell membrane compared to surface cell membranes. Then we have that:

$$\lambda = \frac{p_s}{p_b} = \frac{\beta_b}{\beta_s}$$

Now using our functional form for β_T, we find:

$$P_{\text{burst}} = \frac{\beta_{surf}}{\alpha}\left(1 - \frac{1}{\lambda^2}\right)$$

We can note that making the deep sea cell stronger (increasing λ) gives only a small improvement in the achievable pressure. Thus, we can get a fair idea of where the critical depth is by taking $\lambda = 10$ (the cell membrane can withstand 10 times higher pressure differences) and plug in the fitted values for β_{surf} and α to obtain:

$$P_{\text{burst}} \approx 4200 \text{ atm}$$

We conclude then that any life on Phorcys existing beyond a depth of 20000 meters will need to be radically different than life on earth. If it has a cell membrane, that membrane must be several orders of magnitude stronger than a typical earth cell membrane, and in particular, one can use the expression for P_{burst} above to determine how strong it would need to be.

Beyond 33,000 metres, we expect the pressure of the water to cause a phase change to solid, as we calculated in section 2.5. In the solid ice, we would expect at most microbes to live, if they could somehow withstand the pressure.

4 Conclusion

From the parameters given and a few assumptions, we've estimated several quantitative parameters for Phorcys. In particular, we found that Phorcys is likely composed of 20% water by mass (as compared with earth's 0.5% water by mass). The water likely covers the planet and is about 5,500 km thick (which is a large portion of the total $\approx 12,600$ km radius of Phorcys). Beyond around 33 km deep, however, the pressure will compress the water into ice. To determine where the water is liquid on the surface, we modelled the average temperature as a function of latitude. We found that if the water has slightly higher salinity than earth, or the length of a day is slightly shorter, liquid oceans form at the equator.

Assuming the presence of liquid water, we've examined what types of life could live at the different depths. On the surface, we expect some earth-like animals with adaptations such as shortened, possibly larger bodies. In the water, we found several interesting cut-offs: beyond 200 km, the lack of sunlight prevents photosynthesis; beyond 4000 km, the pressure prevents fish from surviving; around 7,500 km the water viscosity achieves a local minimum, which could enable and hamper different forms of life;

and beyond 20,000 km, the pressure would likely prevent cellular life. Phorcys, while having relatively limited surface life, could enjoy a vast diversity of sea life as compared to earth.

To improve our understanding of Phorcys, several improvements to our work could be made. An important step would be to resolve the difficulty with the S_0 term in the diffusion equation. Ideally, one would be able to specify S_0 and T_0 and produce a temperature plot consistent with observation on earth. Once this is resolved, more detailed models than our simple eddy model could be used. Accounting for varying reflectivity of the surface based on ice cover could give considerable improvement. In general, the goal should be to produce a model that, from data we can measure about Phorcys, reproduces earths climate. Then when applied to Phorcys we can have some confidence it gives us conditions on Phorcys.

Additionally, better understanding how surface pressure and gravity affects land animals could provide more quantitative results about the shape and behavior of animals. Furthermore, perhaps physical considerations could inform speculation about life that is very different from that on earth that could exist even in the earth-like regions of Phorcys. Lastly, perhaps in the regime below 20 km interesting life could form despite the tremendous pressure with no analog to earth creatures.

Appendices

A MATLAB implemented finite difference method to find temperature variability

```
%%%%%%%%%%%%%%%%%%%% NOTE %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%     Values used to calculate the temperature          %
%     variation on earth were changed as follows:       %
%                                                       %
%       B         -> 1.45                               %
%       A         -> 201.4                              %
%       mean_temp -> 288                                %
%       b         -> alpha * B                          %
%       S_0       -> 458                                %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%number of angles to consider, we consider each latitude
N = 180;

%set some initial condition on the entire surface
temps(1:N) = 250;

%number of iteration to run, we see convergence around 3500 but it's very
%fast so go with 5000
iterations = 5000;

%define constants
mean_temp = 250;

%stefan boltzmann constant
sigma = 5.67e-8;
%correction to black body radiation since it's not a black body
epsilon = 0.83;
% I(x) = A T_0^4 + B T_0^3
A =  epsilon * sigma * mean_temp^4;

B = epsilon * sigma * mean_temp^3;

mean_earth_temp = 288;

% We take the value of B for earth from North's paper
%Using the expansion above gives a similar but not quite the same number.
B_earth = 1.45;

%earth rotational speed / planet rotational speed
%This comes in in the ratio of D_e to D_p
%For simplicity we assume 1
day_ratio = 1;

%This comes from North's paper
alpha = .6;
%b holds the Diffusivity compenent
```

```
b = (alpha / 2) * B_earth * day_ratio^2;

%solar constant
K = 1361;
lambda = .68;
beta = K * (1 - lambda);

%for 1 earth day we have S_0 = 207 to make the mean work
%This is our fitting parameter
S_0 = 207;

%how much does latitude change at each step
step_size = pi / N;

%construct the denominator
denom = B + (2  * b / step_size^2);

S_0_max = 250;

means(1:S_0_max) = 0;

%if force is true we are forcing the fit using S_0
%otherwise it will give us a list of S_0 and the mean surface temperature
%for each S_0. This is used to determine the appropriate S_0.
force = true;

if ~force
    %calculate the normalization factor for the averaging at the end. This
    %will not be affected by S_0 so we can do it before we start looping.
    normalization = 0;
    for m = 1:N
        phi = pi * m / N;
        normalization = normalization + sin(phi);
    end
    %We want to know mean surface temp for each S_0
    for S_0 = 1:S_0_max
        for i = 1:iterations
            for m = 1:N
               %latitude
               phi = pi * m / N;
               %set up the periodic boundary conditions
               if m == 1
                  prev = temps(N);
               else
                   prev = temps(m - 1);
               end
               if m == N
                   next = temps(1);
               else
                   next = temps(m + 1);
               end
               % set up some things we'll need to make it a bit easier
               sum_neighbors = prev + next;
               diff_neighbors = next - prev;
```

```
                   % begin assembling the numerator
                   % m + 1 term
                   numerator = next * ( (b * cos(phi) / (2 * step_size)) + ( b / step_size^2) );
                   % m - 1 term
                   numerator = numerator + prev * ( ( b/ step_size^2) - ( b * cos(phi) / (2 * step_size))
                   %constant terms
                   numerator = numerator + S_0 - A;
                   % flux term
                   numerator = numerator + beta * sin(phi) / 2;
                   %divide by denominator
                   temps(m) = numerator / denom;
                end
            end
            %calculate the mean temperature weighted by radius of earth at this
            %latitude
            t_avg = 0;
            for m = 1:N
                phi = pi * m / N;
                t_avg = t_avg + (1 / normalization) * sin(phi) * temps(m);
            end
            %store this mean temperature
            means(S_0) = t_avg;
        end
        S_0s = (1:S_0_max);
        %plot the mean temperatures so we can see where it intersects the given
        %mean for Phorcys
        plot(S_0s,means)
    else
        for i = 1:iterations
            for m = 1:N
               phi = pi * m / N;
               %set up the periodic boundary conditions
               if m == 1
                  prev = temps(N);
               else
                   prev = temps(m - 1);
               end
               if m == N
                   next = temps(1);
               else
                   next = temps(m + 1);
               end
               % set up some things we'll need to make it a bit easier
               sum_neighbors = prev + next;
               diff_neighbors = next - prev;

               % begin assembling the numerator
               % m + 1 term
               numerator = next * ( (b * cos(phi) / (2 * step_size)) + ( b / step_size^2) );
               % m - 1 term
               numerator = numerator + prev * ( ( b/ step_size^2) - ( b * cos(phi) / (2 * step_size)) );
               %constant terms
               numerator = numerator + S_0 - A;
```

```
            % flux term
            numerator = numerator + beta * sin(phi) / 2;
            %divide by denominator
            temps(m) = numerator / denom;
        end
    end
    lattitude = (1:180);
    lattitude = lattitude - 90;
    plot(lattitude,temps)
end
```

B Mathematica linear fitting

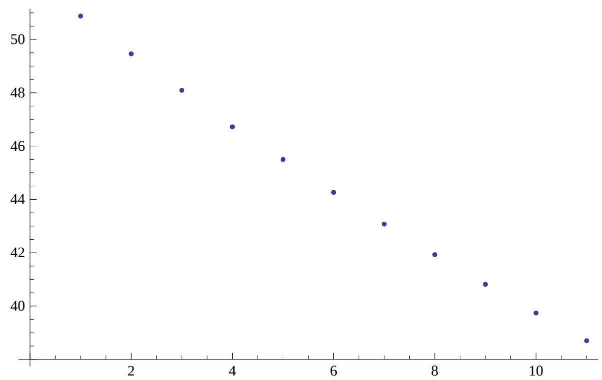
We copy in data obtained in reference [FM] into our notebook:

```
In[12]:= betas =
  {50.879, 49.461, 48.093, 46.722, 45.498, 44.266, 43.076, 41.927, 40.815, 39.739, 38.699};
```

These are measured at T = 0, with the first point measured at P = 1 barr, then P= 100 barr, until P = 1000 barr. We plot these below:

```
In[57]:= ListPlot[betas]
```

Out[57]=



Noticing the approximately linear relationship, we fit a linear function to the beta data:

```
In[58]:= line = Fit[betas, {1, x}, x]

Out[58]= 51.7596 - 1.21485 x

         beta[x_] = 51.76 - 1.21 * x * 1.013 / 100

Out[59]= 51.76 - 0.0122573 x
```

Where the factor of 1.013 / 100 converts x from being measured in 100 barr to measured in 1 atm.

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