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Landauer's Principle in Repeated Interaction Systems

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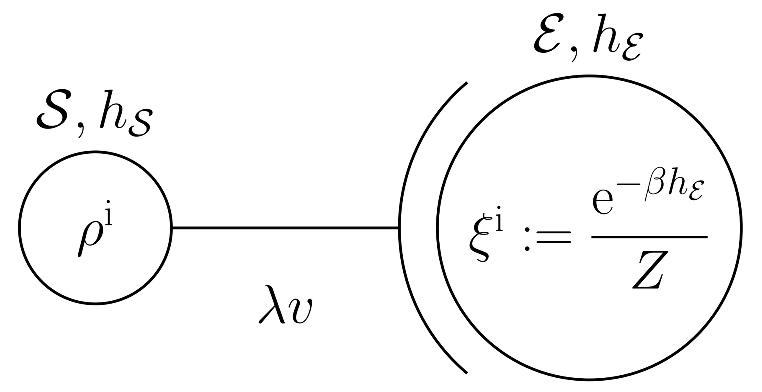
1. Landauer's Principle

A small system $\mathcal{S}$ interacts with an environment $\mathcal{E}$ for time τ:



This process yields, for $U := \exp(-\mathrm{i}\tau(h_{\mathcal{S}} \otimes \mathrm{Id} + \mathrm{Id} \otimes \mathcal{E} + \lambda v))$, final states

$$\rho^{\mathrm{f}} := \mathrm{Tr}_{\mathcal{E}}(U\rho^{\mathrm{i}} \otimes \xi^{\mathrm{i}} U^*), \qquad \xi^{\mathrm{f}} := \mathrm{Tr}_{\mathcal{S}}(U\rho^{\mathrm{i}} \otimes \xi^{\mathrm{i}} U^*).$$

Balance equation: $\Delta S_{\mathcal{S}} + \sigma = \beta \Delta Q_{\mathcal{E}}$.

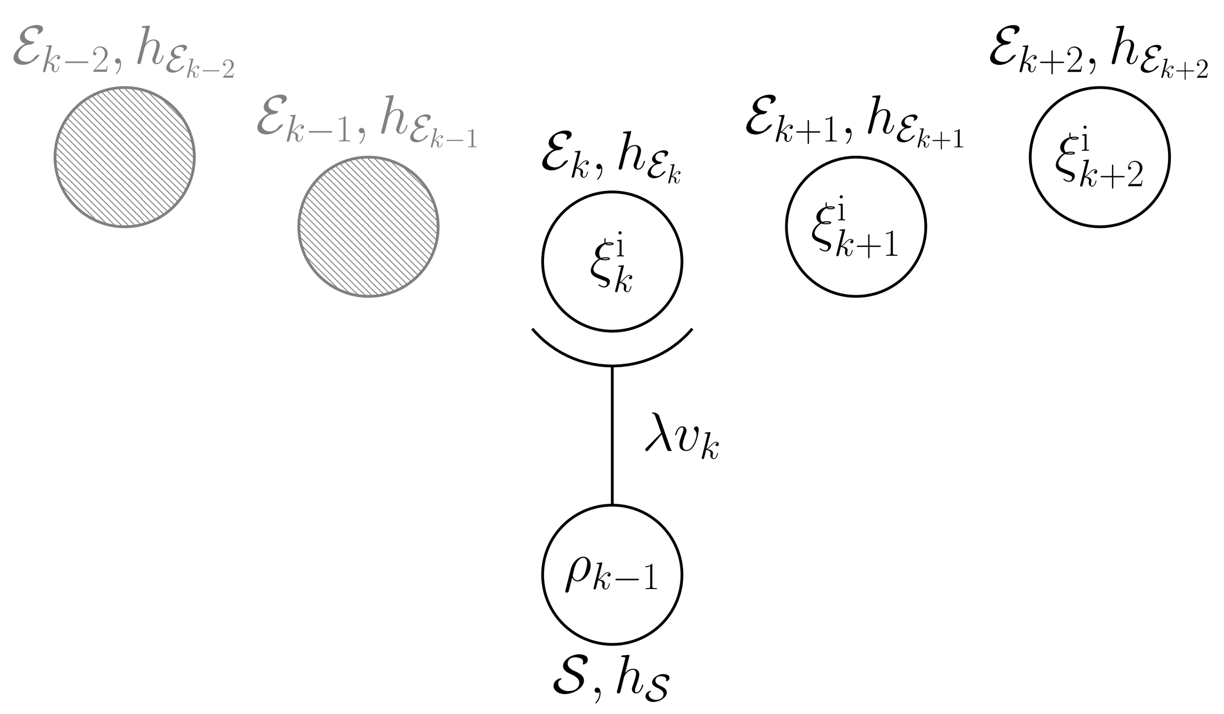
- $\Delta S_{\mathcal{S}} := S(\rho^{\mathrm{i}}) - S(\rho^{\mathrm{f}})$ is the change in entropy of $\mathcal{S}$, where $S(\eta) := -\mathrm{Tr}\,\eta \log \eta$.
- $\sigma := S(U\rho^{\mathrm{i}} \otimes \xi^{\mathrm{i}} U^* | \rho^{\mathrm{f}} \otimes \xi^{\mathrm{i}})$ is the *entropy production*, which we define via the relative entropy, $S(\eta|\chi) := \mathrm{Tr}(\eta(\log \eta - \log \chi)) \geq 0$.
- $\Delta Q_{\mathcal{E}} := \mathrm{Tr}(h_{\mathcal{E}}(\xi^{\mathrm{f}} - \xi^{\mathrm{i}}))$ is the change in energy of $\mathcal{E}$.

The Landauer bound: $\Delta Q_{\mathcal{E}} \geq \beta^{-1} \Delta S_{\mathcal{S}}$.

In particular, the amount of heat exchanged during the erasure of one bit of information is bounded below by $kT \ln 2$.

2. Repeated interaction systems

The system $\mathcal{S}$ interacts with a chain of probes $\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \ldots$.



The *reduced dynamics* on $\mathcal{S}$ induced by the interaction with $\mathcal{E}_k$ is

$$\begin{aligned} \mathcal{L}_k : \mathcal{I}_1(\mathcal{H}_{\mathcal{S}}) &\to \mathcal{I}_1(\mathcal{H}_{\mathcal{S}}) \\ \eta &\mapsto \mathrm{Tr}_{\mathcal{E}}\left(U_k(\eta \otimes \xi^{\mathrm{i}}_k)U_k^*\right) \end{aligned}$$

where $U_k := \exp(-\mathrm{i}\tau(h_{\mathcal{S}} \otimes \mathrm{Id} + \mathrm{Id} \otimes h_{\mathcal{E}_k} + \lambda v_k))$.

Then the state of the system after n steps is $\rho_n = \mathcal{L}_n \mathcal{L}_{n-1} \cdots \mathcal{L}_1 \rho^{\mathrm{i}}$.

3. The adiabatic limit

We consider an adiabatic parameter $T \gg 1$, and a sequence of T probes whose parameters

$$(\beta_{k,T}, h_{\mathcal{E}_{k,T}}, v_{k,T})_{k=1}^T$$

depend on T as follows.

To choose $\beta_{k,T}$ for example, we fix a C^2 function $[0,1] \ni s \mapsto \beta(s)$, and define $\beta_{k,T} := \beta(k/T)$. We choose $h_{\mathcal{E}_{k,T}}$ and $v_{k,T}$ in the same manner.

Thus, $\mathcal{L}_{k,T}$, the reduced dynamics for step k when the adiabatic parameter is T, may be obtained as $\mathcal{L}(k/T)$ for a C^2 function $s \mapsto \mathcal{L}(s)$.

Then, as T increases the parameters change more slowly from probe to probe and we consider more probes overall.

4. The problem

Each step of an RIS satisfies a balance equation so that

$$\sum_{k=1}^T \Delta S_{k,T} + \sum_{k=1}^T \sigma_{k,T} = \sum_{k=1}^T \beta_{k,T} \Delta Q_{k,T}.$$

We are interested in the saturation of the bound, and hence in $\sigma_T^{\mathrm{tot}} := \sum_{k=1}^T \sigma_{k,T}$ in the adiabatic limit:

$$\lim_{T\to\infty} \sigma_T^{\mathrm{tot}} = \lim_{T\to\infty} \sum_{k=1}^T S(U_k \rho_{k-1,T} \otimes \xi^{\mathrm{i}}_{k,T} U^*_{k,T} | \rho_{k,T} \otimes \xi^{\mathrm{i}}_{k,T}).$$

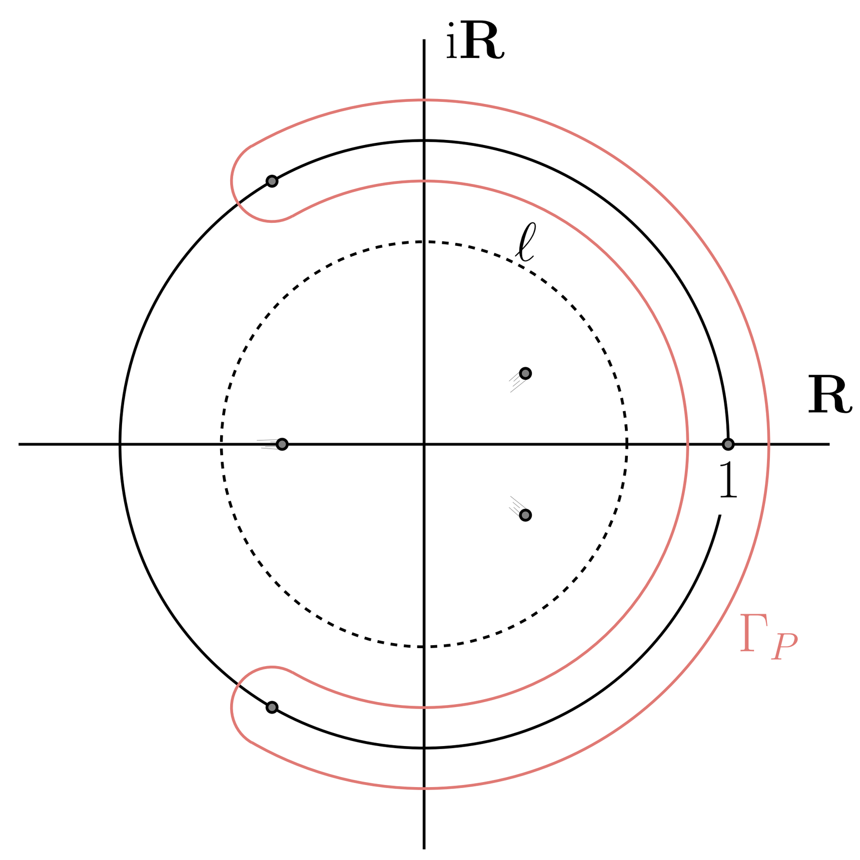
5. Assumptions

We will consider $\dim \mathcal{H}_{\mathcal{S}}, \dim \mathcal{H}_{\mathcal{E}} < \infty$ and make two assumptions.

We assume $\mathcal{L}(s)$ is irreducible for each $s \in [0,1]$.

We define $P(s)$ as the spectral projector of $\mathcal{L}(s)$ corresponding to $\mathrm{sp}\,\mathcal{L}(s) \cap S^1$ and assume

$$\ell := \sup_{s\in[0,1]} \mathrm{spr}[\mathcal{L}(s)(\mathrm{Id} - P(s))] < 1.$$



6. Our answer

Denote by ρ_s^{inv} the unique invariant state for $\mathcal{L}(s)$, and let

$$X(s) := U(s)\left(\rho_s^{\mathrm{inv}} \otimes \xi^{\mathrm{i}}_s\right)U(s)^* - \rho_s^{\mathrm{inv}} \otimes \xi^{\mathrm{i}}_s.$$

- If $X(s) = 0$ for all $s \in [0,1]$, and $\rho^{\mathrm{i}} = \rho^{\mathrm{inv}}_{0,T}$, then $\lim_{T\to\infty} \sigma_T^{\mathrm{tot}} = 0$.
- If $X(s) = 0$ for all $s \in [0,1]$, but $\rho^{\mathrm{i}} \neq \rho^{\mathrm{inv}}_{0,T}$, then $\lim_{T\to\infty} \sigma_T^{\mathrm{tot}} < \infty$, but is possibly non-zero.
- If $\sup_{s\in[0,1]} \|X(s)\|_1 > 0$, then $\sigma_T^{\mathrm{tot}} \to \infty$ in the limit $T \to \infty$.

Eric P. Hanson, Alain Joye, Yan Pautrat, and Renaud Raquépas. "Landauer's Principle in Repeated Interaction Systems". *Communications in Mathematical Physics* (2016). arXiv: `1510.00533 [math-ph]`.